J. Fluid Mech. (1971), vol. 49, part 1, pp. 49–63
Printed in Great Britain

Bed forms due to mass transfer in turbulent flows: a kaleidoscope of phenomena

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(Received 2 November 1970)

This paper gives a brief account and interpretation of bed forms produced during mass transfer from beds of hardened Plaster of Paris (calcium sulphate) exposed to turbulent streams of water. Defects within or upon the initially plane plaster beds perturbed the profile of mass-transfer (solution) rate as the result of the setting up of separated flows. Those defects that exceeded a certain critical dimension set by flow conditions grew in length, breadth and depth into heel-shaped hollows of parabolic plan called *flutes*. The continued growth and eventual interference of coextensive flutes gave rise to a bed with *scallops*, an assemblage of intersecting, saucer-shaped depressions. Defects which fell below the critical dimension set by flow conditions grew initially in length and breadth to give long, narrow *grooves* lying parallel with flow, but after a sufficient time were erased so that the bed reverted to plane. Many plane beds became fashioned into continuous systems of very long *furrows and ridges* aligned parallel with flow and coupled with paired longitudinal vortices resembling Taylor–Görtler vortices kinematically. The dimensions and orientation of this structure suggested that Klineian streaks, which on rigid beds occur randomly in space and time, had become stabilized in space in the new context of the deformable beds. The furrows commonly experienced a higher-order instability which resulted in the production of *secondary flutes* and *scallops* similar to those resulting from pre-existing defects. All of the bed forms described are associated with separated flows, and it is suggested that separation of flow may be a phenomenon necessary for their maintenance and, in the case of flutes, perhaps also for their origin.

1. Introduction

There exist in nature many examples of more or less regularly spaced structures, or bed forms, which arise by the operation wholly of processes of wasting that involve a hydrodynamic interaction, usually complex, between a fluid stream and a deformable solid surface. Well known amongst these structures are the ring waves of glassy australite tektites (Baker 1963; Chapman & Larson 1963), the regmaglypts of metallic and stony meteorites (Nininger 1959; Williams 1959), and the remarkable dune-like waves produced during the melting of ice (Carey 1966; Larsen 1969), all these kinds of surface marking being attributable to ablation. Spelaeologists are familiar with the flute and scallop-like depressions shaped on the walls, ceilings and sometimes floors of limestone caverns as the result of solution (mass transfer) of the rock in underground waters (Curl 1966; Ollier & Trattman 1969). Geologists, too, have observed that the removal of material from stiff mud beds by powerful sand-laden currents (notably turbidity currents) also leads to wave-like, scallop-like or flute-like sculpturings of the surface (Pettijohn & Potter 1964; Dzulynski & Walton 1965). The loss of mass in this case appears due to a combination of scouring by the suspended mineral sand and the direct action of the hydrodynamic stresses exerted by the powerful turbulent stream.

These markings have a remarkable similarity of shape and spatial arrangement and often of size, although produced under widely different conditions in dissimilar materials. However, the similarity is not restricted to geometrical proper ties, for it can be shown that all the structures mentioned, whether due to ablation or mass transfer or sand-blasting combined with the action of fluid stresses, were shaped in association with separated flows that displayed a maximum rate of removal of mass in the vicinity of flow reattachment. Although the mechanisms of wastage are dissimilar, they lead to qualitatively the same bed shapes when separated flows are involved.

The writer has recently completed a monographic investigation into the geological and geomorphological implications of cave flutes and scallops and the erosional flute and scour marks of mud beds. A full account of this work will shortly appear in the geological literature (Allen 1971), but a number of the results obtained, particularly during the simulation of cave flutes and scallops, are thought to be of sufficient interest and novelty in the field of fluid dynamics as to justify a brief description for workers exclusively in this discipline.

2. Experimental method and general development of bed

Cave flutes and scallops, produced in nature by the solution of limestone (chiefly calcium carbonate), were simulated in the laboratory by allowing a hardened bed of Plaster of Paris (hydrated calcium sulphate) to dissolve beneath a controlled, equilibrium turbulent flow of water in a flume. Each bed was cast from a standardized mix in one operation as a layer approximately 6 cm thick in the flume channel, of length 3·66 metres and width 29·8 cm. Water was recirculated over the bed from a large reservoir into which fresh water was admitted and contaminated water allowed to escape, at a constant rate of approximately 0·145 litre s^{-1}. Water depth and flume slope were checked and adjusted at frequent intervals during each run in order to maintain the equilibrium conditions.

It was possible to study as a function of time either the development of the plaster bed as a whole or specific features formed on or introduced upon the bed. Several different means of flow visualization were used to ascertain the flow fields associated with the forms produced on the plaster beds. The markings developed by solution were preserved and recorded in the form of measured profiles, moulds in rubber latex with admixed inert filler, or as moulds in hand-warmed 'Plasticine'. The experiments were carried out at Reynolds numbers between 7950 and 99 400 based on overall mean flow velocity and channel hydraulic radius.

Several different kinds of bed form were produced in the course of the experiments, each type of solution mark being associated with a definite pattern of flow. The different kinds of mark are thought to represent different cases of instability of the soluble bed, and these cases are summarized for reference in figure 1, emphasis being placed on the inter-relation of the cases, the geometry of the bed form, and the type of flow associated with each kind of form.

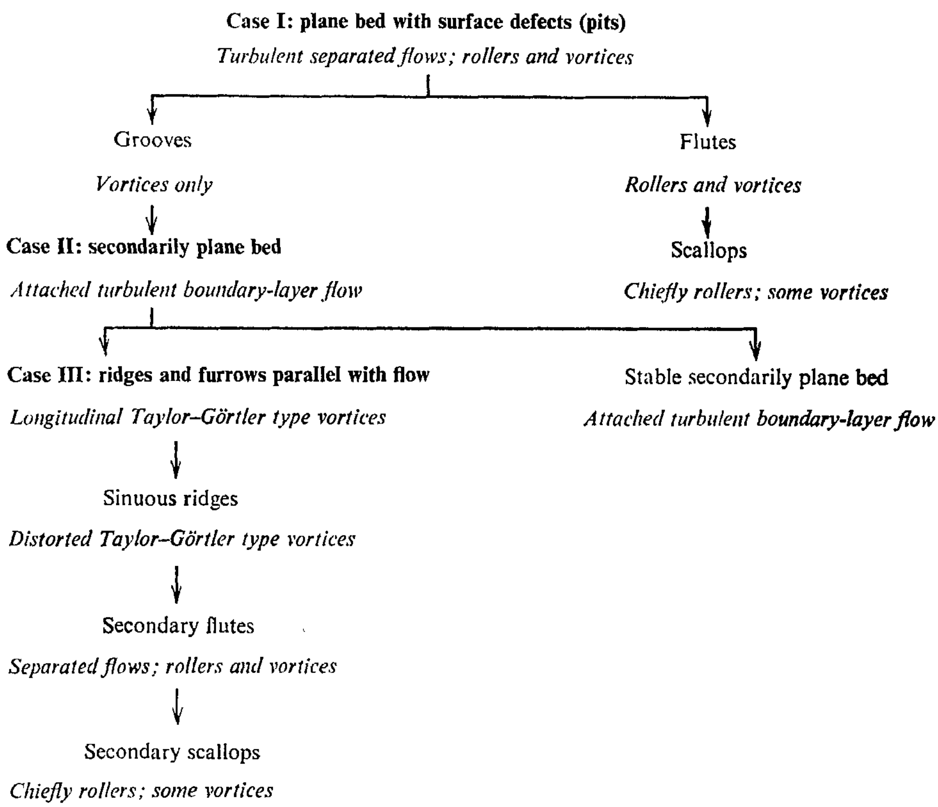

FIGURE 1. Scheme of instabilities, and resulting bed forms and flow fields, associated with mass transfer from Plaster of Paris beds in turbulent water streams.

It is appropriate here to explain some of the terms used in figure 1. Flutes are isolated heel-shaped solutional hollows of parabolic plan-form, whose arms open out down-current. Scallops, produced by the growth of flutes to the stage of interference, are saucer-shaped depressions of polygonal plan. Flutes and scallops are associated with separated flows in which two kinds of kinematic structure are evident: (1) rollers, that is, closed separation bubbles with axes transverse to flow in which fluid elements revolve on closed paths, and (2) vortices, that is, helical spiral flows with axes parallel or nearly so with the external stream. Grooves, associated with vortices, are shallow depressions several times longer than wide and elongated parallel with flow. The ridges and furrows cited under the third case of instability are very many times longer than wide; they are associated with pairs of vortices similar in flow structure, though probably unrelated dynamically, to Taylor–Görtler vortices. The secondary flutes and scallops also cited under the third case are similar to the structures of the same name already mentioned.

3. Flutes and scallops

In a system representing case I (figure 1), we begin with a plane bed carrying a random population of defects, in the form of shallow pits. If some of these defects exceed a critical size determined by flow conditions, the bed proves to be unstable and, after a sufficiently long time of action of the flow, is found to be covered with scallops.

Figure 2 (plate 1) illustrates part of a bed showing a typical assemblage of scallops, the lighting being from the upper left. The photograph was taken after a flow of mean velocity 85·0 cm s^{-1} and hydraulic radius 5·2 cm had acted for 90·2 h on a bed of Plaster of Paris that bore originally on its surface a population of closely spaced defects. During this time, the bed was lowered by solution through a mean vertical interval of 2·92 cm. The scallops are shallow, closely packed, broadly equidimensional hollows, varying in size by a factor of two or three times. The most perfectly developed are polygonal in plan, with 5–7 straight or weakly curved sides. Others are blade or leaf-shaped, and some have a parabolic form. A preferred orientation is virtually restricted to the narrower blade-shaped scallops, which point up-current.

The scallops, whatever their details of shape, were each associated with a separated flow. This was established by mapping on the bed the attitude of small grooves produced at the site of air bubbles entrapped in the plaster (Allen 1966), some of which can be seen in figure 2, and by following fluid marked with dye. The dominant, and in some cases only, element associated with the polygonal scallops proved to be a closed separation bubble or roller. Vortices characterized by helical-spiral internal flow were rarely present, and in any case were relatively small and weak. However, powerful vortices, sometimes to the exclusion of rollers, were held captive by the narrower, blade and leaf-shaped scallops.

The bed forms of figure 2 represent the condition when the vertical lowering of the bed by solution is about 1·5 times the mean spacing in the flow direction of the solutional marks upon its surface. It will be seen that no areas of plane bed remain, for scallops cover the surface completely. A complete coverage by scallops is maintained for all durations of flow such that the value of the ratio of the vertical lowering to the mark spacing is greater than 1·5. For durations of flow leading to smaller values of the ratio, however, the bed is incompletely covered with solutional hollows, and areas of plane bed remain. The flutes then found on the surface become changed into scallops only through a process of increasing mutual interference, as their growth proceeds and the amount of plane bed diminishes relatively. The appearance of the flutes, and their transformation by intersection into scallops, therefore represents the approach to the quasi-steady state of a complete coverage of a surface by scallops.

Flutes became formed in the experiments by the solutional modification and enlargement of defects existing on the plaster surfaces. The process of modification was studied in detail under a broad range of flow conditions by following the development with time of artificial defects of different sizes cut at widely spaced intervals into the surface of a carefully smoothed plaster bed. Each defect consisted of a vertical-sided pit of circular plan with a depth of nominally one-third the diameter. A total of 144 pits of nominal diameter between 0·475 and 2·54 cm was studied. Some pits were changed into flutes that grew continuously in all dimensions with time, whereas others grew in some dimensions over an initial period to become grooves, but were eventually erased in favour of a plane bed.

Figure 3 (*a*)–(*e*) (plate 2) shows a typical series of 'Plasticine' moulds illustrating successive stages in the development of a flute from one of these pits. The pit had an initial diameter $d = 0{\cdot}96$ cm and an initial depth of 0·349 cm. It was acted on by a turbulent water stream of overall mean velocity $U = 45{\cdot}0\,\mathrm{cm\,s^{-1}}$ and hydraulic radius 4·76 cm. The overall mean bed erosion velocity during the course of the experiment was $V = 3{\cdot}47 \times 10^{-6}\,\mathrm{cm\,s^{-1}}$. If t seconds is the total duration of flow to the end of a given stage of the experiment, the solutional mark at that stage may be characterized by the non-dimensional time Vt/d.

The pit contains a separated flow and from its upstream rim sheds vortices into the current to form a wake extending a considerable distance downstream from it. The earliest observed effect of the flow is a rapid solutional attack upon the downstream face of the pit, namely, the surface facing the current, and upon the bed beneath the wake. As the result of this attack, a long narrow groove is speedily hollowed out downstream from the pit, which itself undergoes at first a slight enlargement in the upstream and lateral directions (figure 3 (*a*), (*b*)). By a somewhat later stage, however, the groove has a length many times the diameter of the original pit, and little trace of the pit is to be found (figure 3 (*c*)). Lateral furrows can be observed on the flanks of the almost perfectly symmetrical mark, and in the median dividing plane a faint ridge is often noticeable. After this stage, the mark, which is now properly a flute, grows apparently indefinitely in length parallel with the stream, in maximum breadth, and also in depth (figure 3 (*d*), (*e*)). A study of 144 pits divided between four experiments showed that the non-dimensional length of the marks increased approximately as $(Vt/d)^{\frac{1}{2}}$, the non-dimensional breadth roughly as $(Vt/d)^{\frac{4}{5}}$, and the non-dimensional depth approximately as $(Vt/d)^{\frac{2}{3}}$. Details of these relationships, including graphs, are given in the monograph referred to (Allen 1971).

Grooves several times longer than wide are formed from defects smaller than the critical size set by flow conditions, whereas the flutes just described result from defects of a size larger than the critical. The earliest stages in the development of grooves resemble those shown in figure 3 (*a*)–(*c*), that is, there is an initial growth in length and breadth. However, the depth of a mark which becomes a groove is found to decrease for *all* times. Eventually, length and breadth also begin to decrease with increasing time, and the mark, at the commencement of this stage resembling that seen in figure 3 (*c*), becomes changed into a pointed groove (figure 3 (*f*)). With the further elapse of time, the upstream end of the groove opens out laterally to an increasing degree and the depth progressively decreases. A groove at an advanced stage in this process is shown in figure 3 (*g*). Finally, the groove disappears and the bed reverts to plane. The grooves shown in figure 3 (*f*), (*g*) were grown from a pit of diameter 0·99 cm and depth 0·342 cm, acted on by a flow of overall mean velocity 15·0 cm s^{-1} and hydraulic radius 4·76 cm. The overall mean bed erosion velocity during the experiment was $1{\cdot}03 \times 10^{-6}$ cm s^{-1}.

The critical defect diameter for the production of flutes, made dimensionless by reference to the flow hydraulic radius, appeared to vary with flow conditions as $Re^{-\frac{7}{8}}$, where *Re* is the Reynolds number based on hydraulic radius and the overall mean flow velocity.

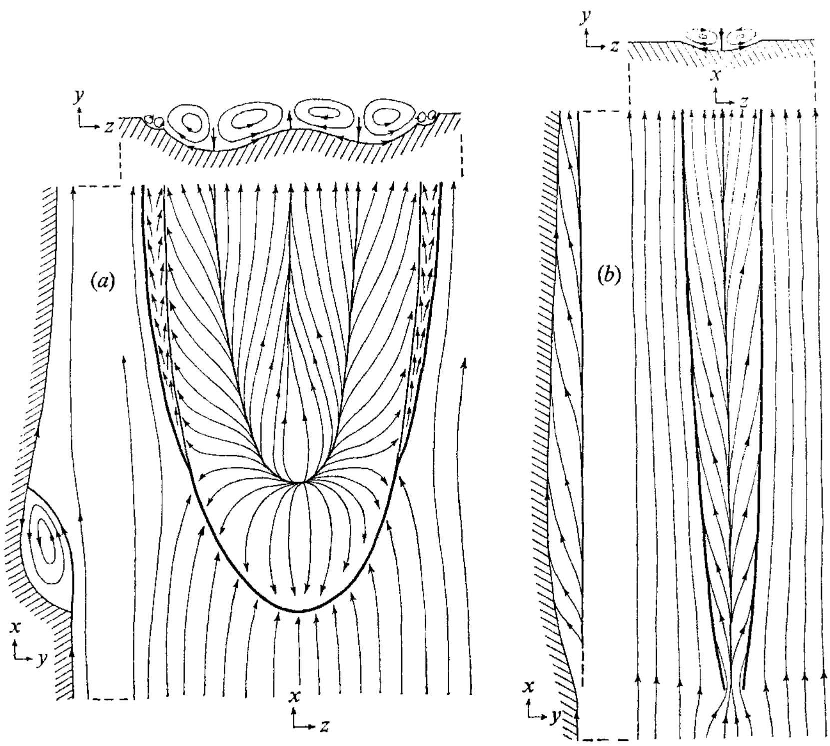

FIGURE 4. Schematic flow fields (skin-friction lines and streamlines) associated with (*a*) mature flutes and (*b*) mature grooves.

The flutes and grooves shown in figure 3 were all associated with separated flows. This was established by following the path of marked fluid and by mapping the attitude of small grooves produced at air bubbles trapped in the plaster, some of the latter being visible, for example, in figure 3(*e*). The results may be summarized by the schematic patterns of skin friction lines and streamlines shown in figure 4 for mature flutes and grooves. The upstream part of each flute holds captive a closed separation bubble, or roller (figure 4(*a*)), but along each flank there lies a powerful vortex that rises up out of the mark to pass downstream and blend into the external flow. These vortices commonly each give rise on the side bounded by the external stream to a pair of small vortices occupying a narrow lateral furrow. The median ridge of the flute is generally a zone of weak streamwise separation of flow. The term separation (and correspondingly reattachment) is here being used in a broad but widely understood descriptive sense. There is a striking concentration of vorticity in the flow associated with the flutes. The flow field associated with mature grooves is much simpler (figure 4 (*b*)), consisting of a pair of oppositely rotating and relatively weak streamwise vortices reattaching to the bed on the floor of the mark. These vortices become progressively weaker with increasing time and, of course, disappear when the plane-bed condition is attained. There is some evidence of vorticity concentration in the case of the grooves.

4. Longitudinal ridges and furrows

Commonly all or very nearly all of the defects originally present on the experimental plaster beds had dimensions smaller than the critical, so that the entire bed, or at least very substantial portions of it, became secondarily plane. Most of these beds became involved in another mode of instability, shown as case II in figure 1, whereby streamwise ridges and furrows were developed on the surface.

Figure 5 (*a*) (plate 3) shows a rubber latex mould of an assemblage of longitudinal ridges and furrows that covered a secondarily plane bed in the flume from wall to wall. The overall mean flow velocity was $29{\cdot}3\,\mathrm{cm\,s^{-1}}$ and the hydraulic radius measured 5·49 cm. The bed had been exposed to flow for 193 h, during which time the surface was lowered through a mean vertical interval of 2·09 cm.

The ridges and furrows are long, almost perfectly rectilinear structures with a remarkably regular transverse spacing which is a tiny fraction of the streamwise length. In one experiment a set of furrows was traced over a streamwise distance of 87 cm, and furrow lengths of a few decimetres proved common. In twelve experimental runs, up to a Reynolds number of 69 700, the mean transverse spacing of the furrows and ridges measured between crests ranged between 0·71 and 1·05 cm. Transverse profiles obtained using a vernier microscope showed that the ridges and furrows varied in shape, but not in size, according to the degree of maturity of the assemblage, that is, the duration of the erosional process. These profiles are shown schematically in figure 6 (*a*). In immature assemblages, the ridges and furrows are subdued features of about equal curvature in transverse profile. As their age increases, the ridges become sharper crested while the furrows grow more rounded, the relief of the surface growing bolder in harmony. In mature assemblages, for example, the ridges are sharply cusped and their transverse spacing is between 5 and 10 times the depth of the furrows. No change in transverse spacing is, however, observed in the course of these modifications.

The orientation of tiny grooves generated in the plaster by entrapped air bubbles gave a major clue, when examined under a binocular microscope, as to the flow field associated with the ridges and furrows, and suggested an explanation for the structures. Figure 6 (*b*) shows schematically the skin-friction lines that may be deduced from the pattern observed. The grooves make angles projected onto a horizontal plane generally between 5 and 10 degrees relative to the streamwise elongation of the ridges. The orientation of the grooves shows that the near-bed flow is directed symmetrically toward a ridge and again symmetrically away from streamwise lines drawn along the deepest parts of the adjoining furrows. The ridge crests may therefore be interpreted as streamwise separation lines, and the longitudinal axes of the furrows as lines of streamwise reattachment. The bed features would appear to be associated with paired vortices similar in kinematic structure to the Taylor–Görtler type, as sketched in figure 6(*c*), the transverse distance between any adjacent pair of ridges corresponding to one wavelength of the vortex system. Under favourable circumstances, as in figure 5(*a*), the vortex system may stretch across the full width of flow and occupy large areas of the bed.

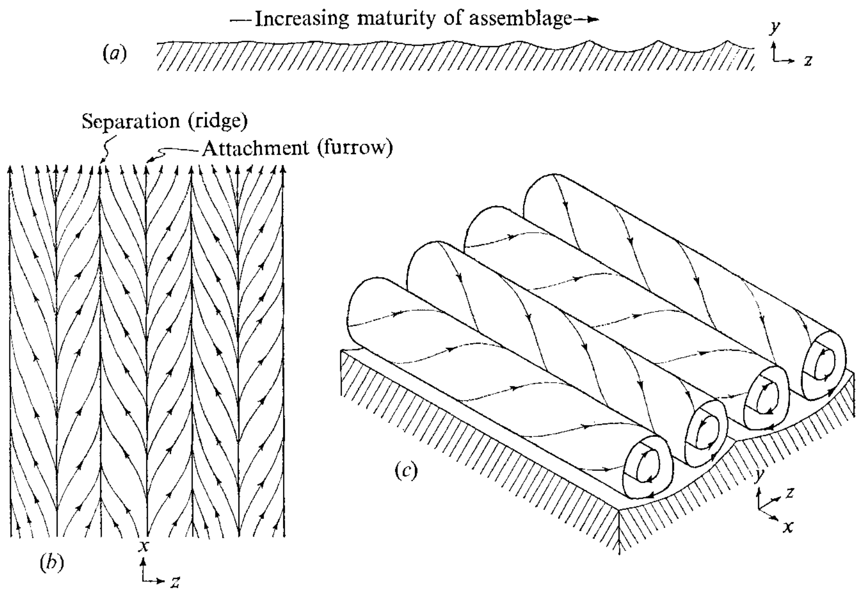

FIGURE 6. Features of ridges and furrows. (*a*) Changes with time in transverse profile. (*b*) Schematic pattern of skin-friction lines. (*c*) Three-dimensional reconstruction of flow field.

The assemblages of ridges and furrows, of which that shown in figure 5(*a*) is typical, were generated by flows of relatively large hydraulic radius. If secondary flows of the type known to occur during turbulent motion through sharp-cornered channels (e.g. Prandtl 1952) had created the markings, one would have expected for the structures a transverse spacing of about 10 cm, which is an order of magnitude larger than the spacing observed. Evidently, secondary flows of this type cannot explain the ridges and furrows.

A tentative explanation of the markings may, however, be advanced in the light of what is now known of the fine-scale structure of the turbulent flow over a smooth or nearly smooth boundary, such as is represented by the secondarily plane beds of the present experiments. Kline *et al.* (1967) showed that, in turbulent flows over smooth rigid boundaries, there is a fine-scale 'streaky' near-bed kinematic structure, the elements of which have, for *brief* periods, a strong resemblance to the *steady* secondary flows believed to be associated with the ridges and furrows. On a *rigid* boundary, the elements of the structure, that is, the

Klineian streaks, arise randomly in space and time. Now if turbulent flow took place over a substantially smooth but *deformable* boundary (e.g. a Plaster of Paris bed) with some initial inhomogeneities to ensure that vortex-like motions recurred at fixed places, one should find that the streaks eventually became locked onto the bed and therefore fixed in space, only the transverse dimension being set by flow conditions. Geometrical features which reflected the kinematic structure and spacing of the steadied streaks should then gradually form on the bed. In terms of their shape and associated flow structure, the ridges and furrows produced on plaster seem to be consistent with a system of steadied Klineian streaks, and therefore may depend on the streaks. This conclusion would be further strengthened if quantitative agreement existed between the spacing of the ridges and the streaks.

Kline *et al.* (1967) showed that the mean transverse spacing of the streaks was directly proportional to the fluid kinematic viscosity but inversely proportional to the shear velocity. Using the empirical transverse spacing obtained by spectral analysis, a numerical factor of approximately 150 appears in the relationship between mean spacing, viscosity and shear velocity. Further, Kline's reconstruction of the flow field of the streaks strongly suggests that the streak spacing corresponds to the wavelength of the vortices associated in figure 6 (*c*) with the ridges and furrows. Hence we may tentatively compare the measured values of the mean transverse spacing of the ridges on the plaster beds with the calculated mean spacings of the Klineian streaks that would exist under the same flow conditions. As was noted, the mean ridge spacing varies between 0·71 and 1·05 cm. The calculated mean streak spacing for the same experiments ranges between 0·56 and 0·97 cm, on the basis of Kline's data using spectral analysis. The discrepancy between the figures in the two sets is 5–20 %, which is sufficiently small to lend weight to the proposed connexion between the ridges and furrows and Klineian streaks.

In a few experiments, made at the lowest Reynolds numbers in the experimental range, the secondarily plane bed remaining after the disappearance of the initial grooves continued to be plane for a relatively very long period, up to the limit of the run. No new features of relief arose and the plane bed appeared to be stable, as indicated in figure 1 under case II.

5. Secondary flutes

The streamwise ridges and furrows described above became involved eventually in a further instability, shown as case III in figure 1. The furrows as the result were modified locally into secondary flutes, which grew in length, breadth and depth with time, and in due course coalesced to give fields of intersecting scallops in the same manner as flutes generated from primary defects.

The onset of instability in an assemblage of furrows and ridges is indicated by the local slight deepening and widening of the furrows and by a change in the plan-form of the ridges from rectilinear to more or less regularly sinuous, the sinuosities, which are weak, having a streamwise wavelength of 5–15 times the mean transverse ridge spacing. Figure 5 (*b*) shows a rubber latex mould of an assemblage of furrows and ridges in an early stage of unstable growth into secondary flutes. The run was made at an overall mean flow velocity of 29·8 cm s^{-1} and a hydraulic radius of 7·32 cm. The current acted for 218 h and the overall mean bed erosion velocity was $2{\cdot}82 \times 10^{-6}$ cm s^{-1}. Inspection of this and similar moulds showed that the change in the shape of the bed was accompanied by a change in the kinematic structure of the near-bed flow. In the widening parts of the furrows, the skin-friction lines diverge more from the mean flow direction than in the un-widened parts, pointing to a locally heightened vorticity concentration.

The further changes in the character of the bed are illustrated by rubber latex moulds shown in figure 7 (plate 4) and by the schematic flow fields sketched in figure 8.

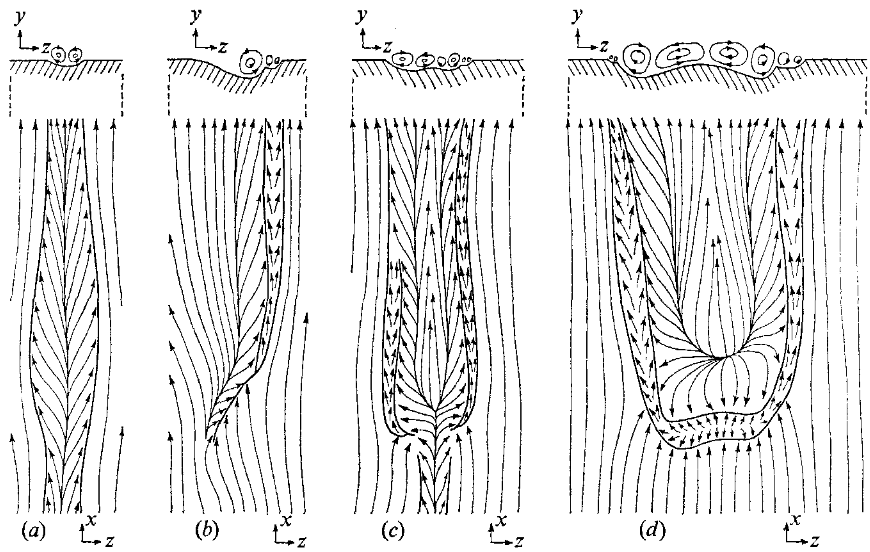

FIGURE 8. Schematic flow fields (skin-friction lines and streamlines) associated with stages in the development of secondary flutes from longitudinal ridges and furrows.

As the furrow widens, the vortices in the widening portion grow steadily larger and more vigorous, to the extent of substantially disturbing, and sometimes considerably suppressing, the flow in the vortices situated laterally. Figure 7 (*a*) shows a mould with two furrows in this stage of development. The one on the left in the picture is growing from a single furrow, while that on the right is being shaped from a pair of narrow, rather poorly-developed furrows. In this case, the overall mean flow velocity was 27·9 cm s^{-1}, the hydraulic radius equalled 1·40 cm, the duration of the run was 140·5 h, and the overall mean bed erosion velocity measured $1{\cdot}62 \times 10^{-6}$ cm s^{-1}. Figure 8 (*a*) shows the kind of flow field associated with furrows at the stage of development of figure 7 (*a*). Often furrows at this stage show a faint median ridge, on which there is a weak streamwise separation of flow persisting for no great distance downstream.

Usually, the furrows widen symmetrically about the flow direction, but in a few cases the enlargement proceeds unevenly, so that an asymmetrical hollow develops on the bed. The right-hand mark in figure 7 (*a*), for instance, is slightly asymmetrical, but a much more typical example is shown in figure 7(*c*). This structure, produced in the same run as the unstable furrows in figure 7(*a*) already described, consists of a groove, rather deep and much longer than wide, lying at a fine angle across the mean flow direction. The tiny striae on its surface, developed from air bubbles trapped in the plaster, point to a flow field similar to that sketched in figure 8(*b*). Instead of a pair of oppositely rotating vortices, only one vortex is now present, the other having become suppressed as the result of the asymmetrical growth.

An intermediate stage in the development of secondary flutes from unstable furrows is illustrated by the two structures on the mould appearing in figure 7(*b*), the structure on the left being the least advanced. The overall mean flow velocity was $29{\cdot}2\,\mathrm{cm\,s^{-1}}$, the hydraulic radius equalled $2{\cdot}57$ cm, and the mould was made after the elapse of 111 h, during which period the overall mean bed erosion velocity measured $3{\cdot}00\times10^{-6}\,\mathrm{cm\,s^{-1}}$. These structures are 3–4 times wider than the parent furrows, and they each possess a well-developed median ridge, as well as lateral furrows. In overall plan, a mark at this stage already shows a strong resemblance to the parabolic flutes illustrated in figure 3(*c*)–(*e*), except that the cusped rim developed at the upstream end of the flank of each mark is not yet continuous across the median line. A reconstruction of the flow field (figure 8(*c*)) suggests that there is a powerful vortex against each flank of the mark, and a pair of weak vortices symmetrically arranged about a weak streamwise separation line on the crest of the median ridge. At the upstream end of the mark, the skin-friction lines of the main vortices make a relatively large angle with the mean flow direction. Clearly, the stage when a closed separation bubble, or roller, can appear is not far distant.

The slow growth toward the median line and eventual fusion of the cusped rims observed to be separate in figure 7(*b*) brings the secondary flutes to an advanced stage of development, illustrated in figure 7(*d*). This structure, produced under the same general conditions as the unstable furrows in figure 7(*b*) but at another station on the bed, is approximately six times wider than its parent furrow. It has a bold median ridge, well-developed lateral furrows, and a weak furrow crossing the upstream end of the mark from one flank to the other. In harmony with the growth toward the median line and eventual fusion of the cusped rims, a captive roller appears in the flow field associated with the mark and progressively strengthens. As appears schematically in figure 8(*d*), the roller occupies the upstream end of the flute-like mark and is followed downstream by vortices, one at each flank of the mark and a pair ranged about the median line. Further growth beyond the stage of figure 7(*d*) leads to the disappearance of the weak up-current furrow and the development of a more strongly flaring shape in plan. The parabolic form of a perfect flute (see figure 3(*e*)) has now been attained.

6. Mechanisms of instability and the role of separation of flow

The bed forms briefly described herein are the product of the wasting by mass transfer of a hard rigid substance, in this case Plaster of Paris, exposed to a weakly solvent turbulent water stream. The broad spectrum of bed shapes that have been noted suggests an equally broad spectrum of flow phenomena, and makes it unlikely that a single mechanism of instability is responsible for the structures. Whatever mechanism is relevant in each case, however, the bed forms have all one feature in common, namely, they are associated with separated flows, though not all of the same type or complexity.

Case I of figure 1 concerned the stability of a plane surface bearing defects with the form of small pits. For each flow condition, defects which exceeded a certain critical size grew into flutes (figure 3 (*e*)) which spread outward over the surface, until they coalesced and a field of intersecting scallops resulted (figure 2). Defects which measured less than the critical size, however, at first developed into grooves (figure 3 (*f*)), but later became erased in favour of a secondarily plane bed. Clearly, the unstable defects are able to perturb the distribution of mass-transfer rate in such a way that they grow continuously in length and breadth and depth, until a meeting with adjacent flutes introduces factors that restrict further changes of size. The stable defects, on the other hand, influence the distribution of mass-transfer rate in such a way as continuously to diminish their own amplitude, the secondarily plane bed without defects being the final result.

Whether the defects experimented with are unstable or not is thought to depend on their dimensions relative to the thickness of the viscous sublayer of the prevailing turbulent flow. If the sublayer is sufficiently thin compared with the streamwise dimension of the defect, it should as a free shear layer become fully turbulent, by vortex generation or by involvement with the turbulent outer stream, within the bounds of the separated flow contained in the defect. The separated flow may therefore be expected to be turbulent throughout. If, however, the streamwise dimension of the defect is sufficiently small compared with the thickness of the viscous sublayer, the free shear layer associated with the separated flow in the defect should remain non-turbulent within the bounds of the defect, and hence the separated flow itself should remain non-turbulent.

The significance of the state of the separated flow in each defect is believed to lie in the difference of the local mass-transfer rate between separation and reattachment points in turbulent as compared with laminar separated flows. Referring to data for spheres (e.g. Linton & Sutherland 1960; Galloway & Sage 1968), about which most is known, the difference in the rates is much smaller (and sometimes non-existent) in laminar flows than in turbulent ones, though the greater rate is generally found at reattachment. If the difference is too small, the defect may not grow in depth faster than the bed round about is lowered by solution, in which case the solutional mark will eventually be erased.

The experiments showed that the critical diameter of defects for the growth of flutes was approximately 7·4 times the calculated thickness of the viscous sublayer existing under the same flow conditions. It is interesting in the light of the proposed explanation of defect stability to compare this result with Sato's (1956, 1959) experimental finding that a laminar free shear layer undergoes transition to turbulence at a constant distance of about 6·5 boundary-layer thicknesses downstream from separation, a similar ratio later being obtained by Becker & Massaro (1968).

The origin of the furrows and ridges (figures 5, 6), making case II of figure 1, must probably be assigned to three-dimensional disturbances. It has been tentatively suggested that these are Klineian streaks, already interpreted in detail by Kline *et al.* (1967), which have become fixed spatially through an interaction with the deformable bed. In this case the flow properties, and hence the local mass-transfer rate, are perturbed transversely to flow. The rate should be least at the crests of the ridges, where the local bed shear stress is least, and greatest at the axes of the furrows, where the local bed shear stress reaches a maximum. The interpretation of the ridges and furrows is therefore similar to that employed in the china-clay technique of flow visualization.

The third case of instability (figure 1, case III) concerned the development of secondary flutes and eventually scallops from the ridges and furrows (figures 7, 8). This seems to be the most complicated instance of all, in which the bed is shaped by the simultaneous action of new two-dimensional disturbances added to the three-dimensional disturbances originally coupled with the furrows and ridges. As a result, the local mass-transfer rate is perturbed longitudinally as well as transversely to flow. The pattern of the bed, as well as the flow field implied by the growing secondary flutes, suggests an instability corresponding to one of the higher-order instabilities of Taylor–Görtler vortices, recently discussed by Wortmann (1969). Again because of the deformable nature of the bed, the new disturbances appear able to assume fixed positions in space, instead of being convected with the flow as in Wortmann's experiments.

7. Comparison with other erosional markings

The scallops described above are indistinguishable in general appearance from scallops produced in natural limestone caverns by the solvent action of underground streams (e.g. Curl 1966; Ollier & Trattman 1969). The size of the natural scallops is, however, unlikely to be determined by simple factors. The experiments here outlined suggest that the distribution in the rock of unstable defects of natural origin (e.g. fossil shells, insoluble or poorly soluble crystals or concretions) may influence the size of cave scallops, as well as promote interactions of the sort that produce secondary flutes and scallops from furrows and ridges.

The experimental scallops and flutes also are similar to structures produced during the ablation of meteorites (e.g. Nininger 1959) and to what geologists call flute marks (e.g. Pettijohn & Potter 1964; Dzulynski & Walton 1965). The latter result from the wasting of mud beds by fast sand-laden currents. The scouring of the mud by suspended grains, in addition to the direct action of fluid stresses, appears to be responsible for the mass-wasting. In many instances, the wasting was localized by defects in the mud bed (e.g. worm borings, shells, cracks), so that structures closely resembling the flutes of figure 3 grew up here and there on the bed. Like the experimental structures, the natural flutes of these mud beds are parabolic in plan, with well-developed median ridges and, commonly, lateral furrows. In other cases, the wasting has proceeded to greater lengths, and an association of the markings with initial defects is no longer obvious. The eroded mud surfaces then closely resemble the field of scallops shown in figure 2. The flutes and scallops of mud beds can be shown experimentally to have been associated with separated flows, whence it appears that wasting by scouring and (or) the action of fluid stresses in such flows is at least qualitatively similar to loss of mass by solution.

The mature longitudinal furrows and ridges shown in figure 5(*a*) have no natural counterparts known to the writer. However, the geometry of the bed at an early stage in the instability of the furrows (figure 5(*b*)) finds a close parallel in certain structures, called longitudinal meandering grooves (Allen 1969), produced under a particular range of conditions when weakly cohesive mud beds are eroded. The geometrical similarity is not only of shape but also one involving the transverse and longitudinal dimensions of the sinuosities. Hence these structures also may depend on stabilized Klineian streaks. Like the furrows of the plaster beds, they are able locally to develop into flutes comparing well with those in figure 3.

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FIGURE 2. Experimental scallops developed in Plaster of Paris. Flow from bottom to top. The portion of the bed shown is approximately 17 cm wide. See text for experimental conditions. Note that the photograph shows the original bed and not a mould (cf. figures 3, 5, 7).

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(*Facing p.* 64)

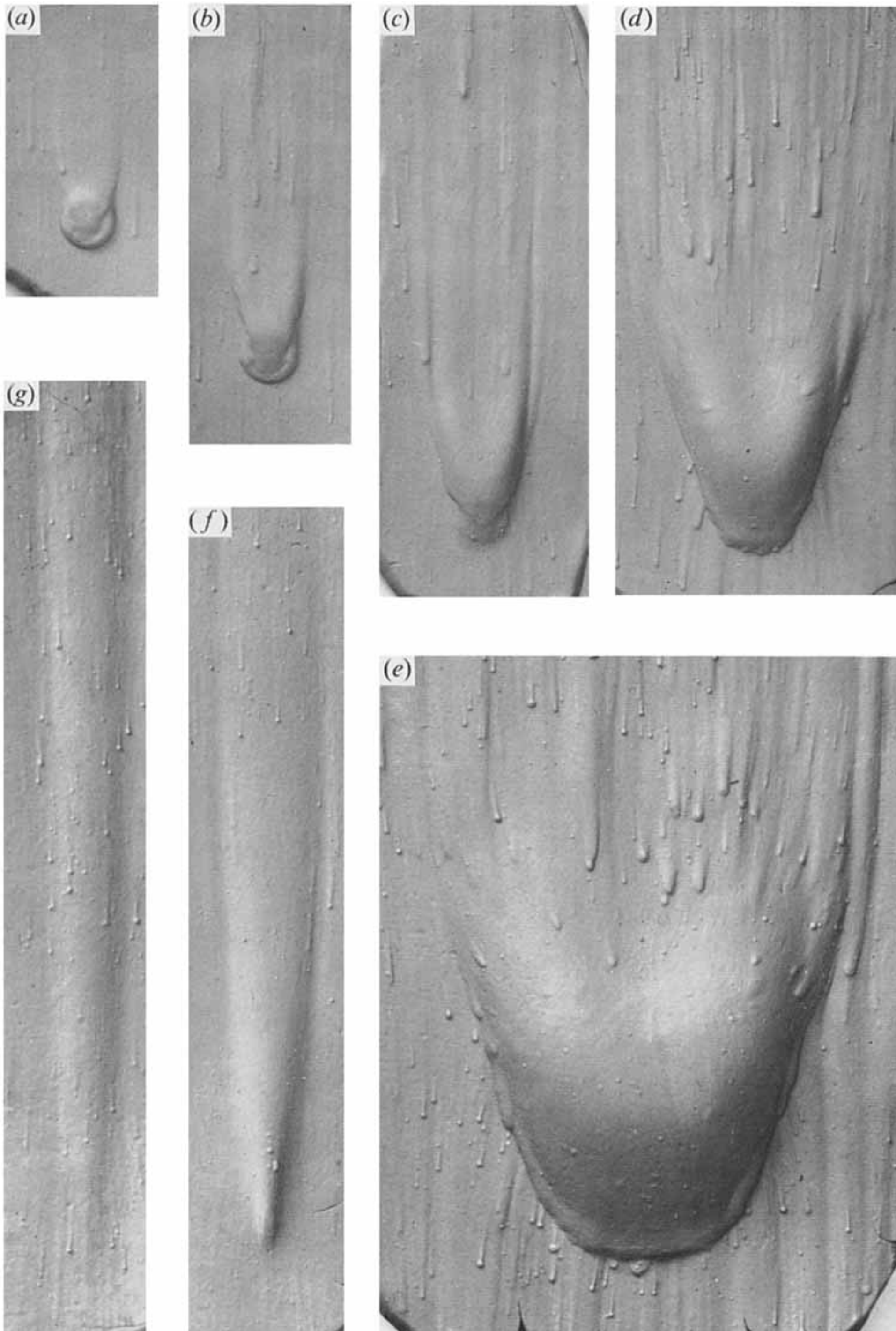

FIGURE 3. Moulds showing stages in the development of experimental flutes (*a*)–(*e*) and grooves (*f*), (*g*) from defects introduced into Plaster of Paris beds. Flow from bottom to top. The portion of the bed shown in (*e*) is 10 cm wide; the other pictures are produced at this same scale. (*a*) $Vt/d = 0{\cdot}066$, (*b*) $Vt/d = 0{\cdot}15$, (*c*) $Vt/d = 0{\cdot}32$, (*d*) $Vt/d = 0{\cdot}74$, (*e*) $Vt/d = 1{\cdot}5$, (*f*) $Vt/d = 0{\cdot}49$, (*g*) $Vt/d = 0{\cdot}98$.

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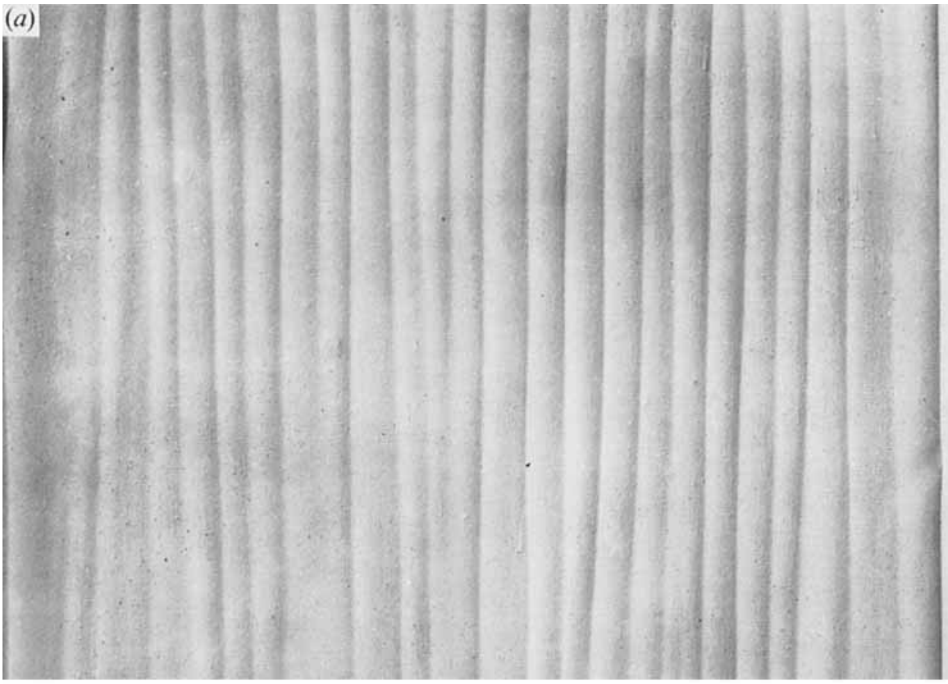

FIGURE 5. Moulds of assemblages of experimental longitudinal ridges and furrows. (*a*) Rectilinear furrows, (*b*) weakly sinuous furrows involved in higher-order instability. In each case flow is from bottom to top and width of bed shown is approximately 28 cm. See text for flow conditions.

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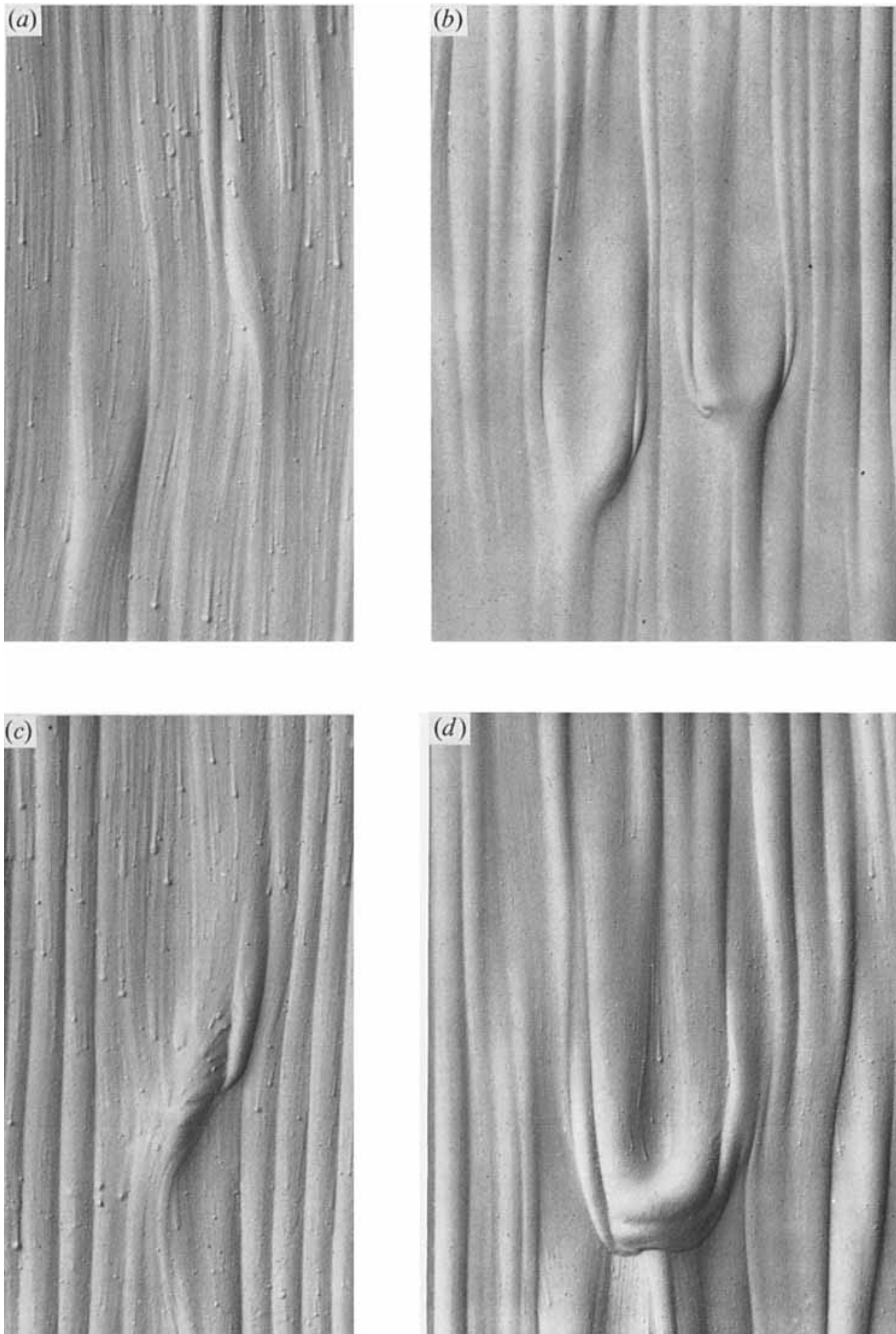

FIGURE 7. Moulds illustrating stages in the development of secondary flutes by the higher-order instability of longitudinal ridges and furrows. Flow from bottom to top. The portion of the bed shown in (c) is 13 cm wide, this linear scale applying approximately to the other moulds depicted. See text for flow conditions.

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